

UNIT 1: FLUIDS

$\rho = \text{density}$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3 = 1 \text{ g/cm}^3$$

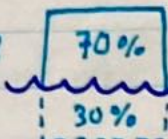
Specific gravity: density of object compared to water

$$\frac{\rho_{\text{object}}}{\rho_{\text{water}}}$$

→ No units (cancel out): just a ratio

→ Specific gravity = percentage of object ^{below} water if floating

$$\text{s.g.} = 0.3$$



→ If $\text{s.g.} > 1$, the object will sink in the water

Fluid: anything that takes the shape of its container [liquids, gasses]

→ Gasses are compressible (variable density), but liquids are not

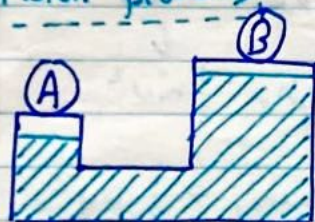
Pressure: force per area [pascals]

$$P = F/A \quad [\text{Unit } \text{N/m}^2 = \text{pascal}]$$

→ Pascal's principle: if a pressure is applied to an incompressible fluid (liquid), the pressure will be evenly distributed in the liquid

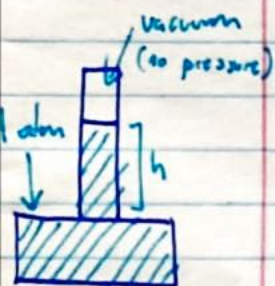
→ Piston problems:

$$P_A = P_B \implies \frac{F_A}{A_A} = \frac{F_B}{A_B} \quad \text{Pascal's principle}$$



$$W_A = W_B \implies F_A D_A = F_B D_B \quad \text{conservation of work}$$

$$F_A/F_B = \text{work ratio (simple machine)}$$



→ Fluids also exert pressure on themselves as a function of depth

If the plane is static, $F_{\text{net}} = 0$, so $F_{\text{up}} = F_{\text{down}}$ and $P_{\text{up}} = P_{\text{down}}$

$$F_{\text{down}} = ma \implies F_{\text{down}} = m_{\text{fluid}} \cdot g = A_{\text{cross-section}} \cdot h \cdot \rho_{\text{fluid}} \cdot g$$

$$P_{\text{down}} = F_{\text{down}} / A_{\text{cross-section}} = h \cdot \rho_{\text{fluid}} \cdot g$$

$$\text{Density: } V/M = \text{m}^3/\text{kg} \rightarrow \rho = V/M$$



Even though pressure comes from the weight of the fluid, it is exerted equally in all directions

IF STATIC: $\text{PRESSURE}_{\text{up}} = \text{PRESSURE}_{\text{DOWN}}$

$$P_1 = P_2$$

$$\rho_1 h_1 g = \rho_2 h_2 g$$

Archimedes Principle: The net upwards force on an object in a fluid is the weight of the displaced fluid

$$F_{NET} = F_{Bottom} - F_{Top} = V_{OBJECT} \cdot \rho_{liquid} \cdot g = mg$$

Flow RATE Measured in m^3/s

$$V_{fluid} = A_{opening} \cdot v_{fluid} \cdot t, \quad v_1 = v_2 \Rightarrow A_1 v_1 \overset{\text{usually the same}}{\boxed{t}} = A_2 v_2 \boxed{t} \quad] \text{ pipe flow}$$

laminar flow

[Equation of continuity: $A_1 v_1 = A_2 v_2$] flow rate must equal

Flux: how much volume crosses an area in a given amount of time?

→ Variable: $R [m^3/s]$

The amount of fluid flowing through a pipe at a given cross-section is always constant, even if the area is not

Bernoulli's equation [conservation of energy]

$$W_i + E_{p,i} + E_{k,i} = W_o + E_{p,o} + E_{k,o}$$

$$\frac{\rho_o m}{\rho} \downarrow \boxed{m} g h_i + \frac{1}{2} m v_i^2 = \frac{\rho_o m}{\rho} \downarrow \boxed{m} g h_o + \frac{1}{2} m v_o^2$$

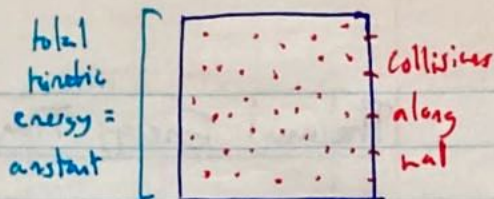
The faster a fluid is flowing, the lower the pressure

$$P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$$

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THERMODYNAMICS

Molecular theory of gases



- Gases are formed by a bunch of molecules bumping into each other
ideal gas = elastic collisions
- When molecules collide with the wall, they change momentum (which exerts a force over time) $F = \Delta p / \Delta t$
- The kinetic energy of all of the molecules stays constant

$$P_1 V_1 = P_2 V_2$$

Pressure · Volume = Constant (Smaller Volume → More collisions → Higher pressure)

→ The temperature of the gas is the kinetic energy of the molecules/molecule

$$T = (\text{constant}) \frac{E_{\text{kin molecules}}}{n} \rightarrow E_{\text{kin}} = \frac{1}{2} n \cdot T$$

$$\frac{P \cdot V}{T} = \text{constant} \rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Universal gas constant
 $= 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}$

IDEAL GAS LAW: $PV = nRT$

absolute 0: temperature when the system has no kinetic energy

Temperature in Kelvin ($C + 273.15$)

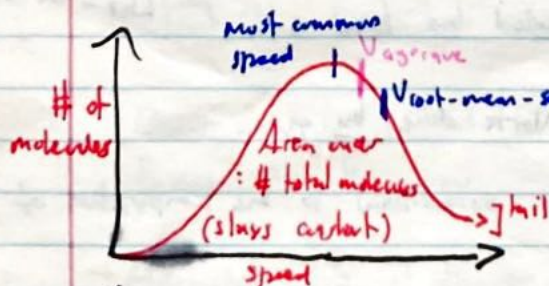
0 K, -273.15°C

→ Mole: molecule count (1 mol = 6.02×10^{23} molecules)

← Avogadro's constant

→ Mass: moles × g/mol element = mass

→ Maxwell - Boltzmann distribution: how fast are gas particles moving?



→ In an ideal gas, ~~all~~ gas particles move along MB distribution

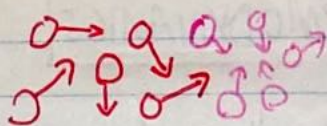
→ Distribution depends on the Temperature of the gas and the mass of the molecules



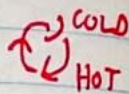
Root-mean-square average:

$$v_{\text{avg}} = \sqrt{\frac{1}{N} (v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2)}$$

Thermal Energy Transfer



Conduction: fast-moving molecules (thermal energy) excite and bump into other molecules, which transfer energy and momentum



Convection: Hot gas is less dense, so it rises (away from the source of heat). Cold gas falls, and becomes heated by the source

Thermal Radiation: Charged particles are getting accelerated (by collisions, etc), making them release EMR

→ Includes the light you see from the heat source

→ Different energy light = different color

Link between Q and $T = Q = mc\Delta T$

(not necessary for physics 2)

Conduction: total kinetic energy stays constant in system, but energy is transferred throughout the system

→ Energy transfer: high energy: low energy (heat)

→ Higher density (closer molecules) = faster heat transfer

power $\left(\frac{J}{s}\right)$ $\left[\frac{Q}{t} = kA \frac{(T_a - T_b)}{d} \right]$ thermal conduction through a barrier
 k = thermal conductivity constant, Q = energy

Laws of Thermodynamics

First law of thermodynamics: conservation of energy

→ Internal energy of gas: translational energy + ^{if gas isn't monatomic} rotational energy + ^{vibrational} oscillation energy

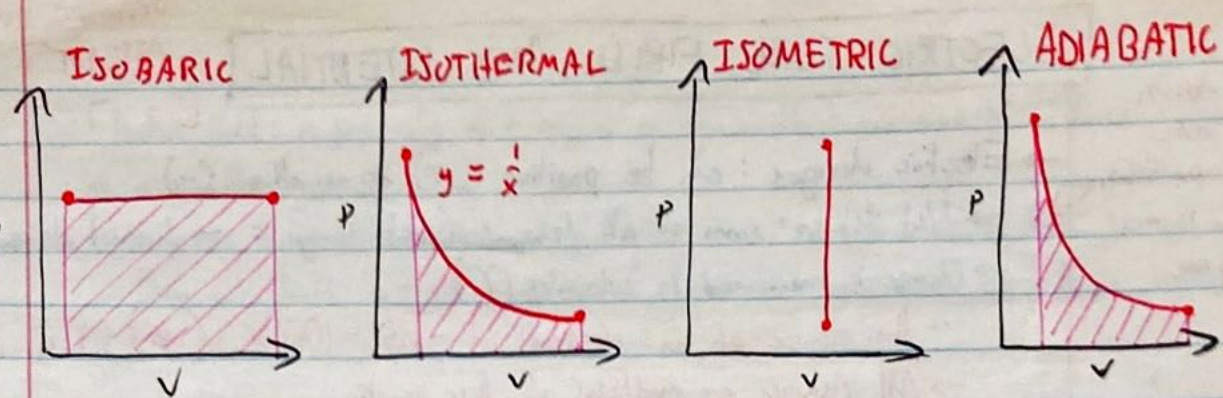
→ $\Delta U = \boxed{Q} + W$ W : adding energy (work done on gas) $\Delta U = Q - W$ W : work done by gas
 can be positive or negative

→ Work done on gas = - Work done by gas

$U \propto T$ internal temperature is proportional to the temperature of the gas

PV Diagram: Relationship with the pressure and volume [think piston]
 Four common processes

Area under curve: work done



- | | | | |
|---|--|---|--|
| <p>Isobaric: constant pressure, variable volume</p> <p>$W = P \cdot \Delta V$</p> <p>$\Delta U = Q + -P\Delta V$</p> | <p>Isothermal: Constant temperature, heat energy must leave gas</p> <p>$y = \frac{1}{x}$</p> <p>$\Delta T = 0 \Rightarrow \Delta U = 0$</p> <p>$Q + W = 0$</p> <p>$P \cdot V$ is constant</p> | <p>Isometric: Volume stays constant (only heat can change energy)</p> <p>$W = 0$</p> <p>$\Delta U = Q$</p> | <p>Adiabatic: No heat is exchanged (but temperature might)</p> <p>No external heat</p> <p>$Q = 0$</p> <p>$\Delta U = W$</p> |
|---|--|---|--|

Kinetic-Molecular Theory of gasses

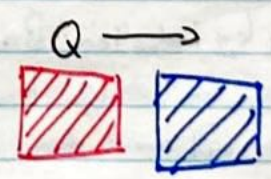
→ Macroscopic properties of gasses (pressure, volume, temperature, etc) are a result of the properties (position, speed, momentum) of each gas molecule

$\frac{3}{2}nRT = \frac{3}{2}Nk_B T = \frac{3}{2}PV = N_{\text{molecules}} \cdot E_{k(\text{avg})} = U$] If monatomic, ideal gas

$E_{k(\text{avg})} = \frac{3}{2} \boxed{k_B T}$

per molecule Boltzmann gas constant

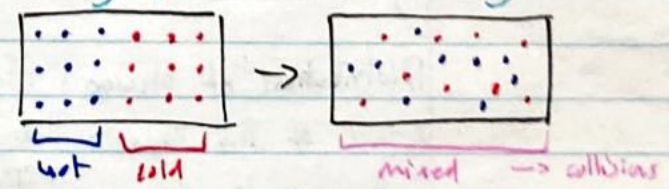
Second Law of Thermodynamics



Entropy can only decrease if work is done on a system

→ Heat flows from hot → cold (spontaneously), but never the other way

→ Order will always tend to disorder, but never the other way



→ Macrostate = lots of disorder (but no specific configuration) | Microstate = exact
 # microstates for macrostate

→ Entropy: S [disorder] $\Rightarrow S = k_B \cdot \ln(W)$

→ A closed system will always tend to maximum entropy

ELECTRIC FORCE, FIELD, AND POTENTIAL

LIKE CHARGES REPEL
OPPOSITE CHARGES ATTRACT

For large objects, this even's out.
For small particles, this charge is a major force

- Electric charges: can be positive (+) or negative (-)
- Net charge: sum of all charges (net charge = - if most charges are negative)
- Charge is measured in coulombs (C)
 - the charge of an electron is $-1.6 \times 10^{-19} \text{ C}$ (e) (1 quantum)
 - All charges are multiples of this quantum
- The charge of an object is determined by the number of electrons
- Conservation of charge: charges cannot be created or destroyed, just moved around

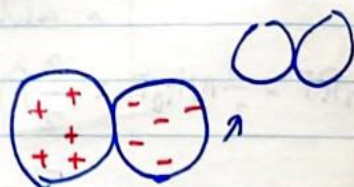
Always some level of resistance (unless it is a superconductor)

- Conductors are made from materials that have weak forces between their electrons and nuclei
 - Metals are good conductors (sometimes metalloids)
- Insulators have strong bonds with their electrons, so they don't conduct electricity
 - Non-metals

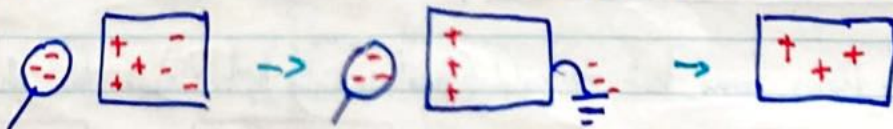
Charging by friction: Rubbing a conductor and insulator together will cause electrons to jump from one object to the other (net charge still 0)

Only the negative charges can move

Charging by conduction: Bringing two charged objects together will re-distribute the charges between them



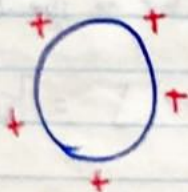
Charging by induction: Bringing a charged object near a neutral object will cause the charge to separate. If a grounding wire is attached, the electrons will move down the wire, leaving a positively charged object



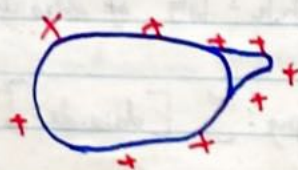
Distribution of charge: On conductors, charges are pushed to the surface of the object. The charges try to be as far away as possible (will move to pointy areas). In insulators, charges are locked where they are



Insulator



Conductor



Conductor

Coulomb law: $F_e = \frac{kq_1q_2}{r^2}$

→ Electric field: a property of a region of space that can exert a force on a specific type of object.

→ Electric fields affect charged particles

→ They can both attract and repulse objects

→ Electric field strength: E (measured in N/C)

How much force is exerted in the field lb. per coulomb

uniform electric field

→ Force in electric field: $\vec{F} = q\vec{E}$ [F is force, q is charge, $\vec{E}$ is field strength]

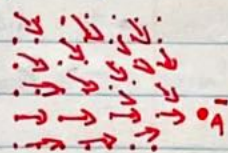
→ Direction of the force is the same as the direction of the field

→ If there are multiple fields, add them as vectors

→ Electric field diagrams can be used to illustrate fields

→ The vectors always point towards $\ominus$

→ Positive particles will follow the arrows, negative ones won't



$F_e = \frac{kq_1}{r^2}$ (field)

← same charge

→ A charged particle in a field can have electric potential energy

→ The amount of energy it takes to keep an object at its position (per charge)

→ Measured in J/C , but the unit volt (V) is used

→ "Zero of potential": area where $V = 0$

→ Potential energy of particle: $\Delta U_e = q\Delta V$

→ Comparison of energy problems: electric potential energy (qV), elastic potential energy ($\frac{1}{2}kx^2$), gravitational potential energy (mgh), kinetic energy ($\frac{1}{2}mv^2$)

→ Potential is scalar, so $-300 V > 200 V$

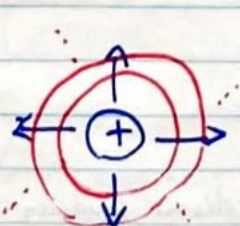
→ In a uniform field, $\vec{E} = \Delta V / \Delta d$ ($\vec{E}$ can also be measured in V/m)

→ A uniform field is like the work-energy theorem: a force must be exerted to move a particle / charged object against the field

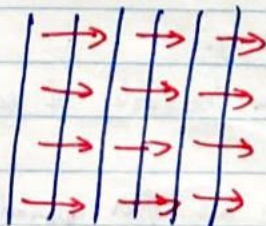
→ Equipotential Line: Lines where a given particle would have the same potential

→ Perpendicular to electric field lines

→ Moving from one line to another takes energy / work

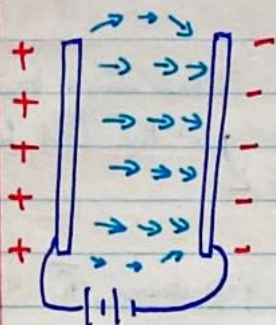


Single charge



Uniform field

→ Special Geometries (specific situations): Parallel plates



→ Between two charged plates: non-uniform electric field

→ Electric field between the plates: $|E| = |\Delta V / \Delta d|$

→ Can be used to make a capacitor, which stores electric charge

→ $\Delta V = Q / C$ [C = capacitance, measured in farads = $\frac{\text{coulombs}}{\text{volt}}$]

→ $C = \kappa \epsilon_0 \cdot A / d$

→ ϵ_0 : vacuum permittivity, value = $8.85 \times 10^{-12} \text{ C/V}\cdot\text{m}$

→ κ : dielectric constant, measures the efficiency of the insulator between the plates

→ $\kappa_{\text{vacuum}} = 1.0$, $\kappa_{\text{air}} = 1.0$ (higher number = better insulator)

→ Point charge: any massless thing producing a charge

Coulomb's constant

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q|}{r^2} \quad \left[q = \text{charge of point, } r = \text{distance, } \frac{1}{4\pi\epsilon_0} = k = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \right]$$

→ Negative → towards negative charge [field]

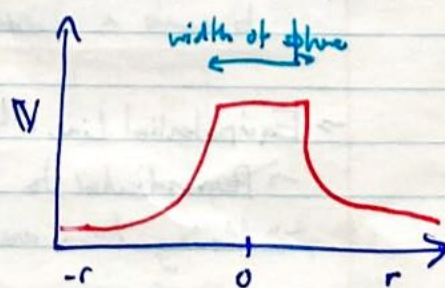
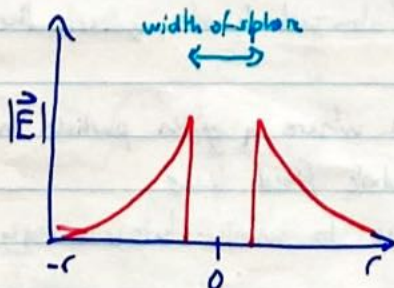
$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad \left[\text{same constants, but } q \text{ can be positive or negative} \right]$$

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \left[\text{force between two point charges} \right]$$

→ Vectors often need to be used to solve these problems

electric field force

→ Conducting sphere: no charge inside of its bounds, no potential difference inside



→ Remember! From the center

→ r should still be from the center of the sphere

→ Kinematics and dynamics are often applied to problems involving electricity

ELECTRIC CIRCUITS

→ **Circuit**: a path along which a charge will flow

→ **Current**: the flow of positive charge (measured in coulombs / second)

→ $I = \Delta Q / \Delta t$ [measured in amperes]

→ A battery is required to form a current (positive and negative terminal)

→ The positive terminal has higher potential, and the negative one has low potential

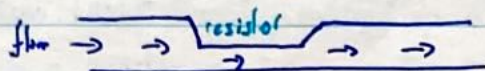
→ The (positive) current flows from $+$ → $-$

→ **Resistance**: property of a circuit that impedes the flow of the current

→ Measured in Ohms (Ω)

→ Affected by the wire material (resistivity = ρ), the length (L), cross-sectional area A

→ $R = \rho \frac{L}{A}$



→ Resistor: something you put in the circuit to change the resistance

→ Ohm's law dictates how resistors affect the current

→ $I = \frac{\Delta V}{R}$] refers to a specific part of that circuit

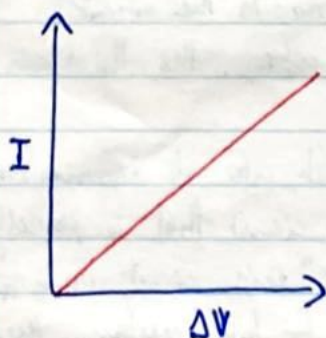
→ When current flows through a resistor, electrical energy is converted to heat

Power = J/s ... → $P = I \Delta V$ [energy dissipated per second] = $I^2 R = \frac{\Delta V^2}{R}$

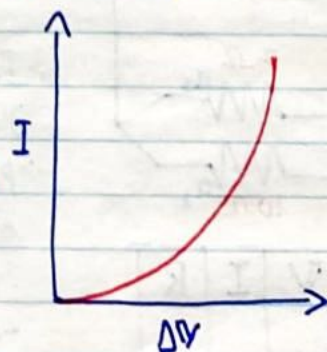
→ Ohmic vs. non-ohmic resistors

→ Ohmic: maintains the same resistance even when the voltage is different

→ Non-ohmic: different resistance for a different voltage

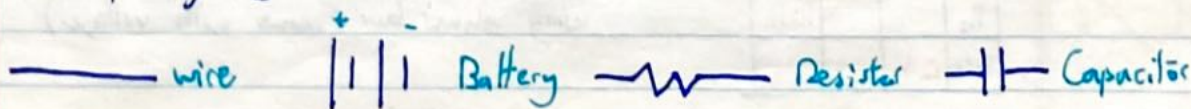


→ Ohmic
→ Resistance = $1/\text{slope}$ (which is constant)

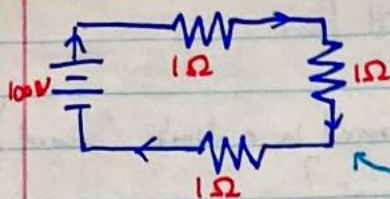


→ Non-ohmic
→ Not a constant slope

→ Circuit diagrams



→ Resistors in series



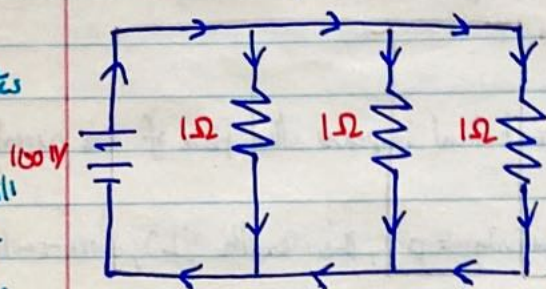
→ Connected in a line, one after another

→ Total resistance = sum of all the resistors

$$\rightarrow R_{\text{total}} = \sum_i R_i$$

$$\rightarrow R_{\text{total}} = 1\Omega + 1\Omega + 1\Omega = 3\Omega$$

→ Resistors in parallel



→ Several possible paths for the current to flow

→ Current splits, goes through the resistor, and joins

$$\rightarrow \frac{1}{R_{\text{total}}} = \sum_i \frac{1}{R_i}$$

$$\rightarrow \frac{1}{R_{\text{total}}} = \frac{1}{1\Omega} + \frac{1}{1\Omega} + \frac{1}{1\Omega} = 3\Omega, R_{\text{total}} = \frac{1}{3}\Omega$$

→ Total resistance is always less than any one resistor

→ Important rules

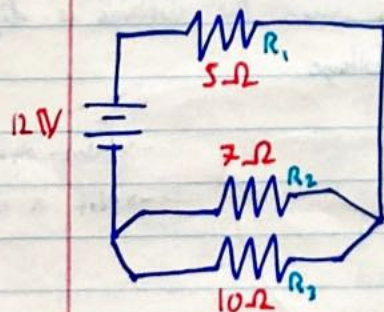
→ For a circuit in series, the charge that flows through the circuit is equal everywhere (resistor 1, resistor 2, resistor n, total circuit)

→ Because of conservation of charge: charge is never destroyed

→ For a circuit in parallel, the voltage that flows through the circuit is equal everywhere (parallel pathway 1, parallel pathway n, total circuit)

→ Because of conservation of energy: for any path, initial and final energy are the same

→ V-I-R charts



	V	I	R
R_1			5Ω
R_2			7Ω
R_3			10Ω
Total	$12V$		

→ A chart that contains a voltage, current, and resistance value for each resistor in the circuit

→ If a row has two entries, the third can be found using Ohm's law

→ Possible steps [first, enter all information]

→ Any piece of the circuit that is parallel can be turned into an "equivalent" series circuit using $\frac{1}{R} = \sum \frac{1}{R}$

→ Then you can solve for total resistance, then current

→ Find places where the current is the same, and solve using Ohm's law (same with voltage)

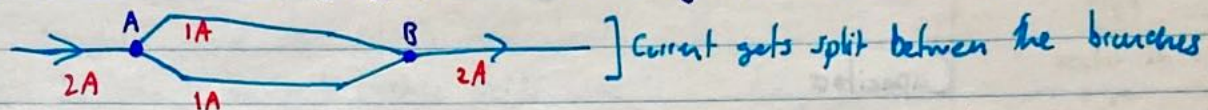
If a parallel segment creates a short circuit, the resistance will be 0 (will not pass through the other resistors)

Electrical potential difference (Voltage) adds up

Charge adds up in each segment

Kirchhoff's rules

- ① The same amount of current that enters a junction leaves it (junction rule)



- Applies conservation of charge: the charge doesn't get destroyed
- equivalent: water flowing through a pipe (water doesn't go anywhere)

- ② The sum of voltages around a closed loop is 0

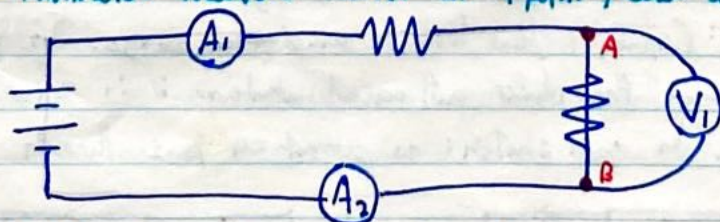
Steps for solving using the loop rule

0. Collapse parallel paths + simplify
1. Choose the direction of the current
2. Follow the loop in that direction, adding up all the voltages
 - a. Voltage of a resistor is $-RI$ [Ohm's law]
 - b. Voltage of a battery is added if it aligns with the direction, and subtracted if it doesn't
3. Set this sum as equal to 0, then solve for I

If you're going against the current, it is equal to $+RI$

→ Circuits from an experimental point of view

- Almost any electronic device acts like a resistor
- The power dissipated is used to light up a bulb, turn a motor, etc
 - ex. higher power = brighter bulb
- Voltmeters measure voltage over a resistor. They are attached in parallel, because voltage is always equal among parallel paths
- Ammeters measure current at a point, and are attached in series



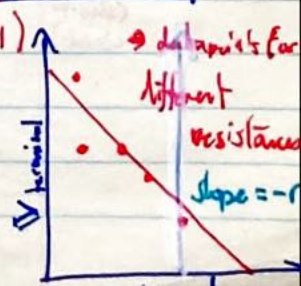
→ V_1 measures potential difference between A and B

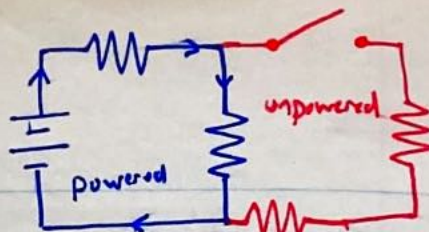
→ Batteries have internal resistance that reduces their voltage



$$\Delta V_{\text{terminal}} = E - Ir = \Delta V_{\text{total}} - (\text{current})(\text{resistance internal})$$

- Terminal voltage depends on the current (and is ohmic)
- By attaching a battery to a circuit and measuring its voltage and the current of the circuit (with various resistors), you can determine the internal resistance of the battery





Switches

- If a switch is open, the part of the circuit attached to it goes dead. It can be completely ignored

Capacitors

- Consist of two metal plates, separated by a "dielectric" (air or something else)
- A charge builds up on each plate (positive, negative), and electric potential energy stored in the field between the plates
- Capacitance: How much charge a capacitor can hold per potential difference

$$C = \kappa \epsilon_0 \frac{A}{d} \quad (\kappa = \text{dielectric constant}, \epsilon_0 = \text{vacuum permittivity}), \quad \Delta V = \frac{Q}{C}$$

- If one plate is $-2C$ and the other is $2C$, $Q = 2C$
- Capacitance is measured in farads (C/V)

$$\text{Energy in a capacitor: } U_c = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

$$\text{Electric field between plates of a capacitor: } E = \frac{Q}{\epsilon_0 A} = \frac{\Delta V}{d} \quad \leftarrow \text{distance}$$

Capacitor circuits

Opposite
of
Resistors

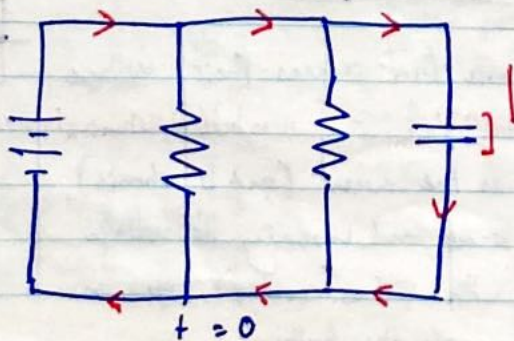
- If capacitors are connected in parallel: $C_{\text{total}} = \sum C_i$ (total capacitance sum of all the capacitors)
- A C - ΔV - Q chart can be used to solve these kind of problems [$Q = C \Delta V$]
- If connected in series, $\frac{1}{C_{\text{total}}} = \sum \frac{1}{C_i}$
- The same strategies from resistors apply (combining parallels, etc)

RC Circuits [Circuits with resistors and capacitors]

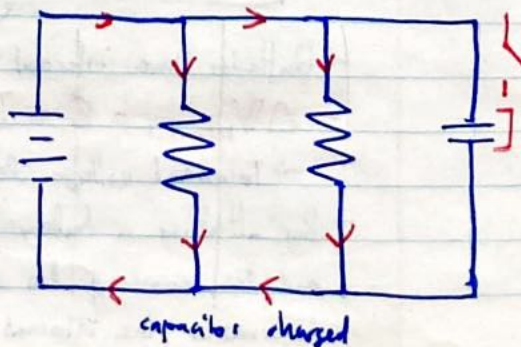
- Will be in one of two states

- ① Capacitor 'just' added to circuit: acts like a wire with no electrical potential across it (charges can flow freely)
- ② Steady state: Capacitor has built to maximum charge. The potential difference between the plates will equal whatever it is in series with it. It acts like an open switch: no current can pass through

(short circuit)

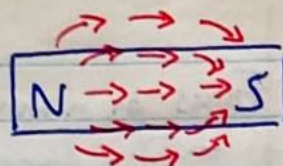


$t = 0$



capacitor charged

ELECTROMAGNETISM

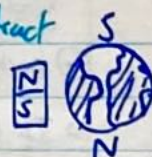


Magnetic fields

- Exist in 3 dimensions (right-hand rules help to visualize this)
- All magnets have a North and South pole
- Some behavior is similar to electric fields
 - Magnets exert forces on each other, have fields that extend infinitely, where strength is inversely proportional to distance
 - Opposite charges/poles attract, like charges/poles repel
- Magnetic field: $N \rightarrow S$ (towards south)
- Magnetic fields created by moving charges
 - Because currents are moving charges, wires create magnetic fields
- Magnetic dipoles will align themselves to a magnetic field (ex. compass)
 - If there isn't a stronger one, it will align to the earth's

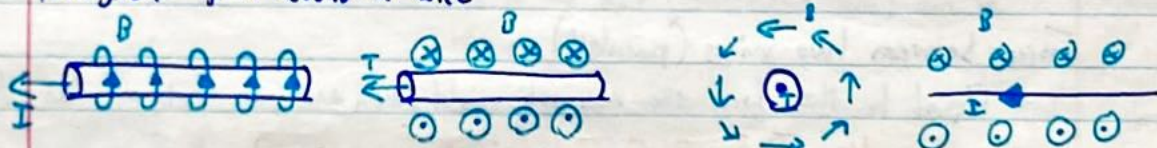
(X)
vector into
page

(O)
vector out
of page

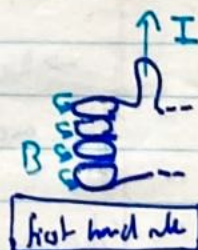


Opposites
Attract

Magnetic field around a wire



- If a current is flowing into the page, the magnetic field will be perpendicular and counterclockwise
- Right hand rule: If you curl your fingers and point your thumb out, your thumb is the direction of the current and your fingers show the magnetic field

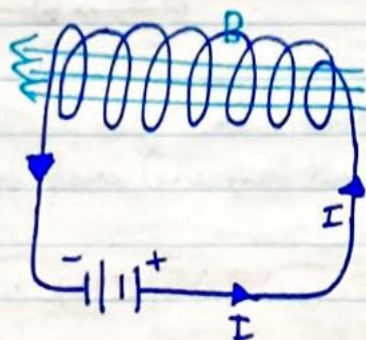


→ Strength of field: $B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$ (μ_0 = vacuum permeability, I = current, r = distance from center of wire)

→ Magnetic field strength is measured in teslas (T) = $N/(A \cdot m)$

→ If a wire with a current is wrapped in a coil shape, it will create a specific magnet / single magnetic field (solenoid) (electromagnet)

magnetic field
in a straight
line (like a
dipole magnet)

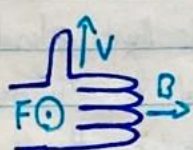


Forces on a moving charged particle

→ If a charged particle moves through a field, it will experience a force

→ $F_m = q\vec{v} \times \vec{B} = |q\vec{v}| \cdot |\sin\theta| \cdot |\vec{B}|$] don't use charge sign

→ Direction is determined by the 2nd right hand rule



⊖ particle [L]



⊕ particle [R]

→ Fingers: magnetic field, Thumb: direction of particle; Palm: force

→ Left hand for negative particles, right for positive

→ Force is just in the opposite direction

→ Everything is perpendicular to each other

→ If $\theta = 0^\circ, 180^\circ$ (particle parallel to field), no force is exerted

Currents are always possible

Wire in a magnetic field

→ Current is always positive = movement of positive charged particles

→ $F_m = I\vec{L} \times \vec{B} = |I\vec{L}| \cdot |\sin\theta| \cdot |\vec{B}|$] L is length of wire

Force between two wires (parallel)

→ Equal to the force one magnetic field from one wire exerts on the other

→ $F = \frac{\mu_0}{2\pi} \cdot \frac{I_A I_B}{r} \cdot L$ [r = distance between wires]

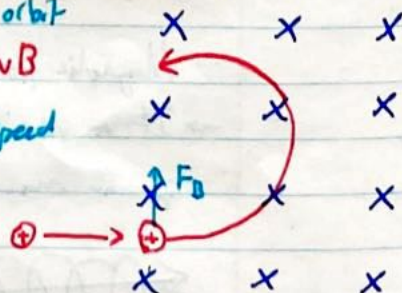
→ Newton's third law: if one wire exerts a force on another, the second wire exerts the opposite force on the other (will repel or attract)

Charges in a magnetic field (mass spectrometer)

→ Because the force is equal to the particle's speed, a charged particle travelling straight into a magnetic field will follow a circular orbit

→ Magnetic force is centripetal force: $F_c = F_B \Rightarrow \frac{mv^2}{r} = qvB$

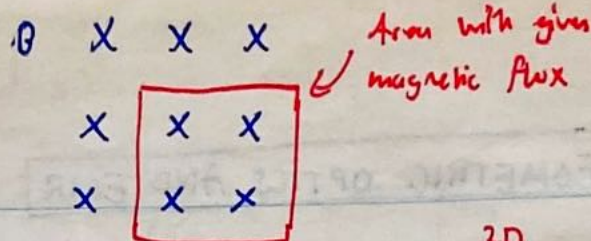
→ If a particle enters the field with some amount of speed parallel to the field, it will follow a helical orbit



Particles moving through multiple fields

→ For the particle to travel in a straight line, the two fields it passes through (usually electric and magnetic) must be of equal magnitude and in opposite directions

→ $F_E = F_m \Leftrightarrow q\vec{E} = qv\vec{B}$ or $F_E = F_g \Leftrightarrow q\vec{E} = mg$ or $F_m = F_g \Leftrightarrow qv\vec{B} = mg$



Magnetic Flux

- Measures the total magnetic field passing through an area (unit: webers wb)
- Magnetic field strength (B) is sometimes called magnetic flux density
- To get field strength from flux, divide it by the total area
- $\Phi = \vec{B} \cdot \vec{A} = |\vec{B}| |\vec{A}| \cos(\theta)$, $|B| = \Phi / (|\vec{A}| \cos \theta)$
- If the area is parallel with the field, $\Phi = 0$ (perpendicular $\Phi = |\vec{B}| |\vec{A}|$)

Electromagnetic Induction

- a current can be induced in a wire by moving it in a magnetic field
- current can be thought of as the movement of positive charges in a wire; the second hand rule can be used to determine the force on the particle (direction of current)
- when the wire is moving perpendicularly to the magnetic field
- The voltage in the wire (electromotive force) can be calculated
- $\mathcal{E} = B l v$ (l = length of wire in field, v = speed perpendicular to field)
- Current can also be induced in a wire loop (and voltage)
- $\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t}$ (N is times the wire loops, Φ is the magnetic flux)
- This voltage can be induced 3 ways:
 - Changing the field strength, changing the loop's area, or
 - changing the orientation of the loop (which changes Φ)

Also applies to rectangular sheet:
 $\mathcal{E} = B l v$
 width of sheet

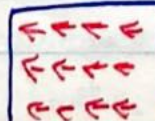
Lenz's Law

- The direction of induced current is opposite to the changes in flux
- the induced current will create a magnetic field that opposes the flux
- the **1st** hand rule can be used to find the current direction
- only applies when the flux of the wire/loop is changing

Work done on a magnet will cause the domains to align

If external field is strong enough → creates permanent magnet

Ferromagnetism



- Have many domains
- Each with a localized $\vec{B}$ field
- Will align, grow, and merge in an external field
- Iron, nickel, cobalt

Paramagnetism

- Domains will align in external field, but result is weak
- No permanent magnets
- Weakly attracted to magnets

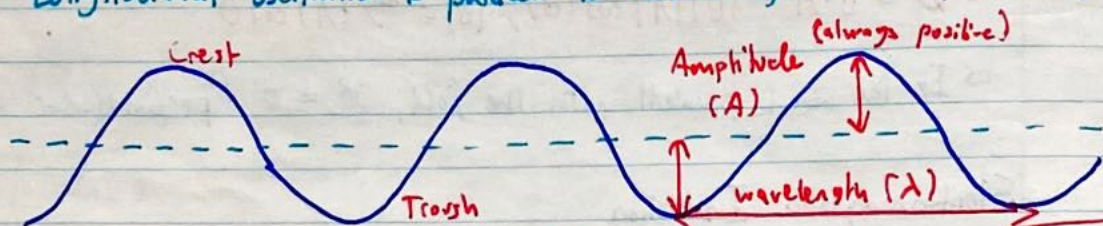
Diamagnetism

- Domains will align opposite to the magnetic field, cancelling part of it
- Weakly repelled by magnets
- Water, graphite

GEOMETRIC OPTICS AND EMR

Waves ^{n of particles}

- A rhythmic oscillation that transfer energy from one place to another
- Transverse: oscillation is perpendicular to direction of motion
- Longitudinal: oscillation is parallel to direction of motion (ex. sound)



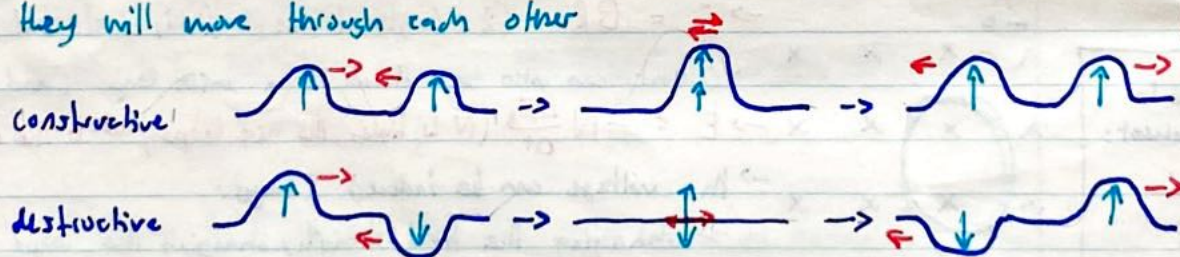
- period: amount of time it takes for the wave to oscillate once
- frequency: amount of times the wave oscillates per second
- wave equation: $x = A \cos(\omega t) = A \cos(2\pi \cdot f \cdot t)$

$$f = \frac{1}{T}$$

$$v = f\lambda$$

Interference

- if waves cross paths, their heights are added to each other, and they will move through each other



Electromagnetic waves (light waves)

- electromagnetic spectrum: range in λ for electromagnetic waves
- gamma rays ($\downarrow \lambda$, $\uparrow f$, $\uparrow E$) → radio waves ($\uparrow \lambda$, $\downarrow f$, $\downarrow E$)
- always travel at the same speed: the speed of light ($c = 3 \times 10^8 \text{ m/s}$)
- created when a charged particle accelerates / oscillates
 - When a charge oscillates, it creates a disturbance in the electric field it creates (which is ~~perpendicular~~ parallel to the movement of oscillation)
 - it also creates a magnetic field disturbance perpendicular to the movement
 - these electric and magnetic fields sustain each other, and propagate in the direction perpendicular to both fields
- Don't need a medium to travel through (disturbance in a field, not in a medium)
- Because all particles above absolute 0 are moving / accelerating, they all produce EMR (and loose energy? if in a closed system)

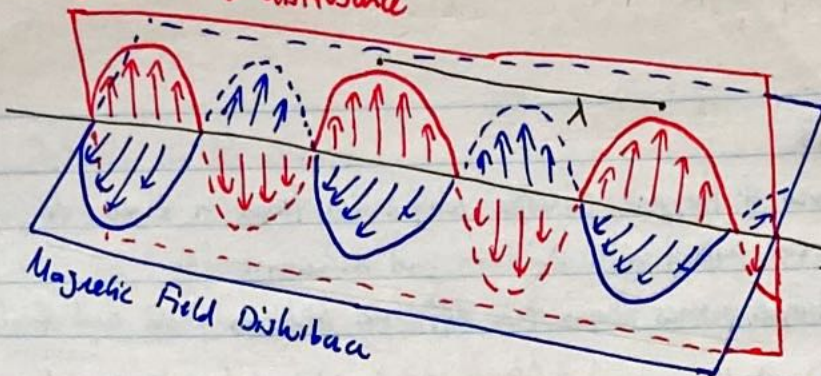
charged ~~black~~ particles create electric fields

Hand rules

white light is made up by a combination of all the other colors

Oscillating charge

Electric field disturbance

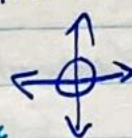


direction of wave propagation

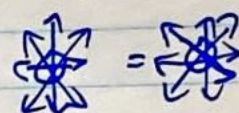
Polarization

- EMR is a transverse wave → it can be polarized
- Polarization: aligning the wave to a certain axis / line
- When light bounces off a surface, the light usually becomes polarized in the plane of the surface
- A polarizing filter only lets polarized light through
- Non-polarized light includes light polarized at every angle
- Circular polarization: angle of polarization changes smoothly

polarized

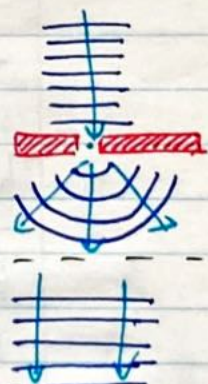


non-polarized



Diffraction, point source model

- Each point on a wave is the starting point of a new wave
- When the wave passes through a boundary, it will "bend" around it: this is why shadows can be fuzzy
- Also why sound travels around corners
- The hole that an object passes through must be about the same size or smaller than the wavelengths for major effects



Double Slits

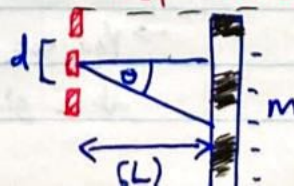
- If light waves pass through two slits beside each other towards (and onto) a screen, the waves will interfere (and the effect will be visible on the area)
- Proves that light is a wave (if it was a particle, two spots would appear)
- Creates a "dotted" interference pattern



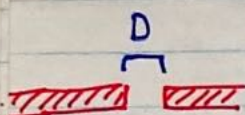
→ Equation: $d \cdot \sin \theta = m\lambda$ (d = distance between slits)

$m = \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}$
 $m = 1, m = 0, m = 1, m = 2$

- Diffraction gratings have many slits, not just two. They produce sharper interference patterns with thinner maxima



Diffraction gratings cause the second order maxima to disperse into a rainbow of colors (because different wavelengths of light diffract at different angles)



Single Slits

- Because of Huygen's principle, diffraction from a single slit will cause a diffraction pattern with minima and maxima
- Interference comes from the difference between each end of the opening
- $D \cdot \sin \theta = m\lambda$ (D is the height of the slit, m refers to minima as 1, 2 etc.)
- Compared to the double slit, the central maximum is very bright, and the others dim

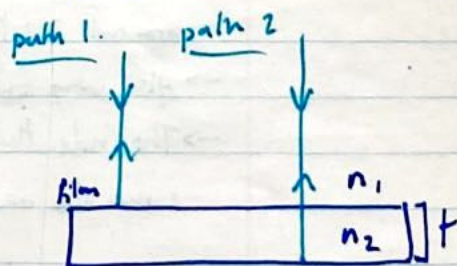
Index of refraction

- Light slows down as it passes through different materials:
- For example, light travels slower through glass than air
- "Speed of light" refers to the speed of EMR in a vacuum
- This difference is measured by the index of refraction (n)
- $n = c/v$ (v is speed of light in whatever medium)
- because $v = f\lambda$ and f stays constant, a light's wavelength also changes when refracted (higher index = shorter wavelength)
- $\lambda_n = \lambda / n$ (λ_n is wavelength in medium, n is index of refraction)
- Different wavelengths get refracted at different angles (more later)



Thin Films

- A thin film over a surface can cause interference because the light can reflect off of different surfaces and overlap with itself
- If the film has a different refractive index (which it will) the amount of wavelengths the light travels through the film depends on its index of refraction

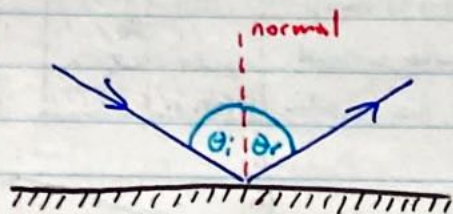


for high → low
index of
refraction

- If wavelengths travelled in film % $\lambda = 0$, constructive interference
- If wavelengths travelled in film % $\lambda = \frac{1}{2}$, destructive interference
- ~~Wavelengths~~ Wavelengths travelled: $m = 2t/\lambda_n \rightarrow 2t = m\lambda_n$
- If going from low → high, a phase shift occurs and the conditions are reversed
- Count the phase changes (even = whole numbers → constructive)
(odd = whole numbers → destructive)
- You see color because only one wavelength can interfere constructively at a given thickness

Wave Behavior at Boundaries

- ① Light can be reflected back off the medium
- ② Light can transmit (refract) through the medium
- ③ Light can be absorbed, which will transmit energy to the (opaque) medium

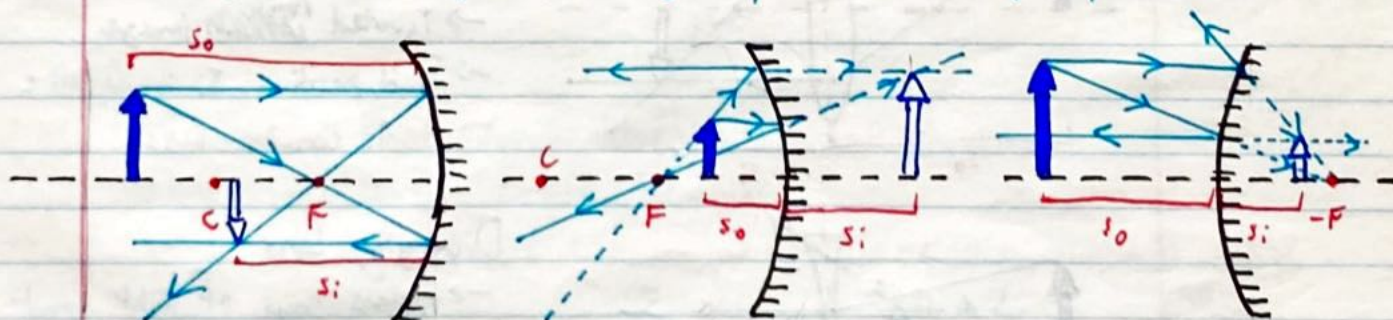


Planar Mirrors

- Light reflects at the same angle that it is incident ($\theta_i = \theta_r$)
- virtual image, $m = 1$, $s_i = s_o$

Optics Terms

- s_i = ^{distance} distance of image from mirror, s_o = distance of object from mirror
- h_i = height of ~~object~~ image, h_o = height of object
- magnification: $m = |h_i|/|h_o| = |s_i|/|s_o|$
- virtual image: image created in the mirror (right side up)
- real image: image created by light rays meeting/being reflected (inverted)
- focal point: for a spherical mirror, point of focus
 - $f = r/2$ (r = radius of mirror) $\& \frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$ ↖ can be projected onto a screen
 - rays parallel to the principal axis reflect through the focal point
 - rays through the focal point reflect parallel to the principal axis

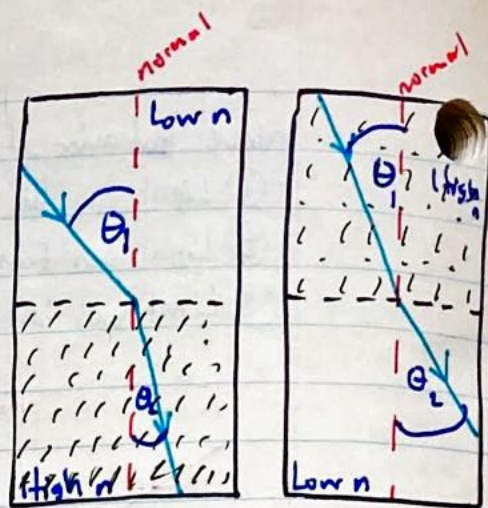


- | | | |
|-------------------------------------|----------------------------------|---|
| → inverted, real image | → upright, virtual image | → upright virtual image |
| → s_o, s_i , and f are positive | → f and s_o are positive | → s_o is positive |
| | → s_i is negative (other side) | → s_i and f are negative (other side) |
| → converging mirror | → converging mirror | → diverging mirror |
| | → magnification > 1 | → magnification < 1 |

Lensmaker's equation: $\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$

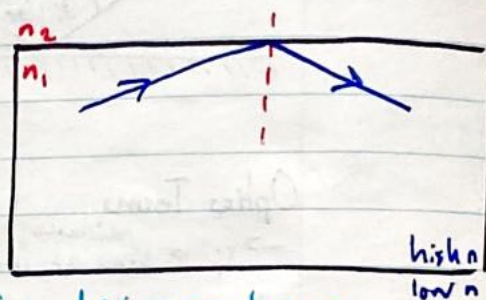
Snell's Law

- Light will change direction during refraction
- $n_1 \sin \theta_1 = n_2 \sin \theta_2$
- low $n \rightarrow$ high $n \rightarrow$ light towards normal
- high $n \rightarrow$ low $n \rightarrow$ light away from normal
- This happens because of the point-source model and the difference in speeds between the media



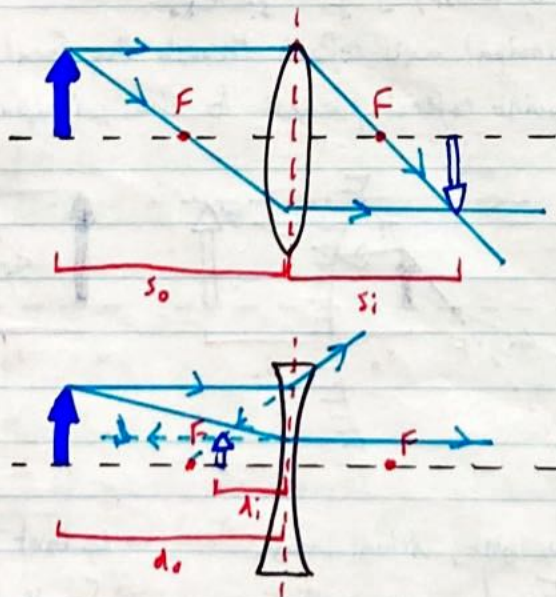
Critical Angle

- At a certain angle, 100% of the "refracted" light is not transferred between media
- Called "total internal reflection"
- happens when a ray of light tries to move from high $n \rightarrow$ low n
- Equation: $\sin \theta_c = \frac{n_2}{n_1} \rightarrow \theta_c = \sin^{-1}(\frac{n_2}{n_1})$



Lenses

- Made from a curved piece of refractive material (like glass)



Converging Lens

- Brings Rays of light together
- inverted ~~real~~ image
- S_o is positive, S_i is negative
- AKA. convex lens

Diverging Lens

- Pushes rays of light apart
- right side up virtual image
- S_o and S_i are positive
- AKA. concave lens

Lenses in General

- the higher the index of refraction of the lens, the stronger the effect of the diverging lens.
- similar to mirrors, but lenses have light passing through instead of being reflected
- Lenses have both a positive and negative focal point

QUANTUM, ATOMIC, NUCLEAR PHYSICS

Special Relativity

SpaceTime and the speed of light

- ① The laws of physics are the same in any uniformly moving reference frame
- ② The speed of light is always measured constantly, no matter the reference frame
 - ↳ This means that time and space aren't constant (!)
 - ↳ The faster you go, the slower time moves and the shorter distances become (in the direction of your travel)
 - ↳ However, you won't notice these effects, only an observer will (because of ①)

Sub-atomic particles

Proton (p)	$1.67 \times 10^{-27} \text{ kg} = 1 \text{ amu}$	$+e = 1.6 \times 10^{-19} \text{ C}$
Neutron (n)	$1.67 \times 10^{-27} \text{ kg} = 1 \text{ amu}$	0
Electron (e)	$9.11 \times 10^{-31} \text{ kg}$	$-e = -1.6 \times 10^{-19} \text{ C}$
Photon (γ)	0	0

→ neutrinos (ν) and quarks, which make up the other particles, were discovered later

Electron-Volt

- Unit of energy useful for atomic physics
- Amount of energy it takes to change the potential of an electron by 1 V
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Photons

- A bundle of EMR (can be thought of like a light particle)
- Photons can have (kinetic?) energy:

$$E = hf = \frac{hc}{\lambda}, \quad h = \text{planck constant} = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$$

- Basically, higher frequency = higher energy (smaller λ , higher energy)
- Radio waves have the least energy, gamma rays have the most

Radio Waves	Microwaves	Infrared	Visible light	Ultraviolet	Gamma Rays
$\lambda = 1 \text{ m}$	$\lambda = 10^{-2} \text{ m}$	$\lambda = 10^{-5} \text{ m}$	$\lambda = 10^{-6}, 10^{-7} \text{ m}$	$\lambda = 10^{-8} \text{ m}$	$\lambda = 10^{-12} \text{ m}$

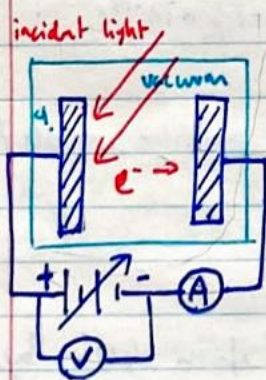
remember
 $c = f\lambda$

Photoelectric Effect

- Proves that light is a particle (what the heck?), or rather, a wave-particle duality
- If light is shined on a metal, it causes electrons to pop off the metal
 - Because a light "particle" is colliding with an atom, freeing the electron
- These can generate a current; the voltage of the circuit can be used to determine the maximal kinetic energy of the ~~photo~~ electrons ($E_{K_{max}} = qV$)
- Conservation of energy applies, but it takes energy to remove an electron from a nucleus (+/- attraction). This is called the work function (ϕ), which is different for each metal

$$E_{K_{max}} = qV = hf - \phi \quad (q \text{ and } V \text{ refer to the electron, } h \text{ and } f \text{ to the photon})$$

Photoelectric Experiment



- Plate A is coated with the metal being studied, and the plates are ~~photo~~ connected to a variable potential source
- The incident light causes electrons to flow (a current), which is measured by $\textcircled{A}$
- The potential is against the electron flow; it is adjusted until the electrons stop flowing (stopping voltage)
- This lets us find the max kinetic energy ($E_K = qV$), which we can compare with $E_K = hf - \phi$ to find the work function
- A given metal has a threshold frequency: light below this frequency won't induce the photoelectric effect
- Brighter light = more photons = more electrons (if over threshold frequency)
- Photons can't "gang up" to knock out one electron
- On an $E_{K_{max}} - f$ graph, the slope of a line (for a metal) is planck's constant, the y-intercept is the work function, and the x-intercept is the threshold frequency

Photon Momentum

- Like a particle, if a photon hits another particle, energy and momentum are conserved
- Derived from $E = hf$ and $E = mc^2 \rightarrow p_{photon} = \frac{hf}{c} = \frac{h}{\lambda} = \frac{E}{c}$
- When photons collide and change paths, this is called scattering
- Because f can change after a collision, the light may change color

De Broglie Wavelengths

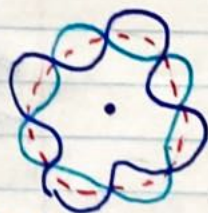
- Waves can act like particles → particles can act like waves (w/ part 2)
- The higher the frequency (lower wavelength), the more particle properties are exhibited (ex. momentum). The lower the frequency (higher wavelength), the more wave-like properties (ex. interference, diffraction)
- Wavelength of a particle: $\lambda = \frac{h}{p} = \frac{h}{mv}$ (m and v refer to particle)

Wavefunctions

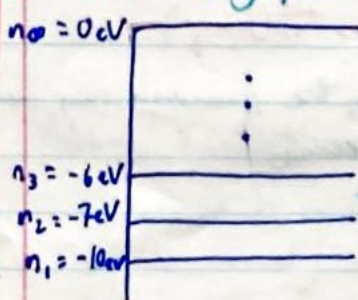
- Electrons are very small: they show wave-like properties, like interference
- When fired through slits (diffraction grating) they produce a pattern similar to light
- The wave of a particle is a function (Ψ) of location: the higher the amplitude of the wavefunction at the location, the higher chance that a particle is to be found there

Energy Levels

- Electrons don't orbit atoms like particles; they form standing waves around the nucleus
- They can only exist in locations where they interfere with themselves constructively (unstable if otherwise)
- These are the energy levels of the atom; each one has a specific amount of energy it takes to free the electron entirely
- Electrons can move between energy levels (or ionize) by absorbing or releasing photons (E_{photon} must equal the energy difference between the levels ^{exactly})



Energy level
is always
negative (takes
this much
energy to leave)



→ Ground state: first energy level (most energy required to remove from atom because of electrical attraction between electron and nucleus)

→ Higher energy level: easier to separate from nucleus (higher distance between them)

$$\rightarrow E_{\text{photon}} = E_{\text{final level}} - E_{\text{initial level}}$$

→ If an electron moves between levels that aren't adjacent, any possible jump between the levels may happen

→ ex. $E_4 - E_1$; $E_4 - E_3$, $E_4 - E_2$, $E_4 - E_1$, $E_3 - E_2$, $E_3 - E_1$, $E_2 - E_1$

→ If an electron receives more energy than necessary to leave the atom, the excess energy becomes kinetic energy

Conservation of...
of nucleons momentum
charge mass/energy

Nuclear Decay

→ "Particles" that leave the nuclei of atoms

→ Alpha (α): two protons and two neutrons (same as helium nucleus)

→ Beta (β^+ or β^-): an electron or a positron (e^- and e^+)

→ Gamma (γ): a gamma ray photon

daughter
nucleus: left over
nucleus
after decay

→ Nuclear notation: ${}^A_Z \text{Symbol}$ (A = total nucleons/mass number, Z = number protons/charge)

Alpha Decay

→ When a nucleus emits an alpha particle (ex. ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^4_2\alpha$)

→ The biggest particle decay, weakest type of radiation

Beta Decay

→ Can emit a positron (β^+ decay) or an electron (β^- decay)

→ These particles have very small mass: the mass number doesn't change

→ However, charge must be conserved, thus:

→ β^+ decay turns a proton into a neutron and emits a positron

→ β^- decay turns a neutron into a proton and emits an electron

→ The charge of the daughter nucleus is different, but the mass is the same

→ Beta decay also emits neutrino (β^+) or antineutrino (β^-) ($\bar{\nu}$)

→ Doesn't affect the decay products, just the total E_K

→ ex. ${}^{19}_{10}\text{Ne} \rightarrow {}^{19}_9\text{F} + e^+ + \bar{\nu}$ or ${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + e^- + \bar{\nu}$

Gamma Decay

→ Emits a gamma ray of light (ex. ${}^{238}_{92}\text{U} \rightarrow {}^{238}_{92}\text{U} + \gamma$)

→ Though the nucleus and charge don't change, the gamma ray carries away energy and momentum, so the nucleus recoils

Neutron Decay

→ A neutron is ejected (${}^{13}_4\text{Be} \rightarrow {}^{12}_4\text{Be} + n$)

→ Creates a different isotope of the same element

Conservation of mass-energy

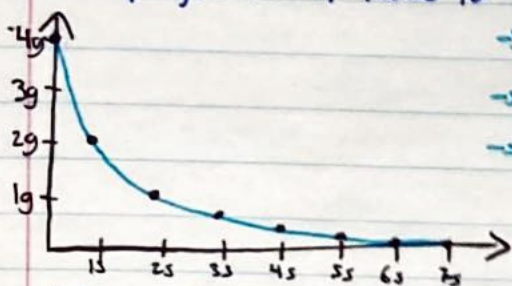
→ The kinetic energy after a nuclear reaction comes from mass ($E = mc^2$)

→ Mass defect (Δm): difference in mass between reactants and products

→ Mass of reactants is higher than products, mass-energy is conserved

Half-Life

- Radioactive isotopes decay into new ones at different rates
- Because of the randomness involved, this rate is an inverse-exponential curve
- Half-life: time it takes for the mass of the sample to decrease by $\frac{1}{2}$



- Can range from centuries to a fraction of a second
- Short half-life = fast decay rate = more dangerous
- Long half-life = slow decay rate = less dangerous

$$\rightarrow m(t) = m_0 \left(\frac{1}{2}\right)^{t/t_{1/2}} : m_0 = \text{original mass} \\ t_{1/2} = \text{half-life}$$

- Measured with a Geiger counter (measures the # of detected decay particles)

Other Nuclear Reactions

- Fission: A large nucleus is split into pieces, liberating energy from mass
 - A neutron is shot at the nucleus to start the reaction
- Fusion: Two small nuclei are combined into a larger (more stable) nucleus
 - A lot of energy is released (large mass defect)
 - Creates a lot of light and heat (the sun uses nuclear fusion)

- Induced Reaction: Nucleus bombarded with high-speed particles
 - Used often as experiments to find new particles

- Antimatter annihilation: particles and anti-particles destroy each other
 - Every particle has an "anti-particle" with an opposite charge
 - When they meet, they release energy in the form of photons (two so that momentum can be conserved)

?
two photons?

$$\rightarrow E = mc^2 \rightarrow E_{\text{particle}} + E_{\text{antiparticle}} = 2E_{\text{particle}} = 2m_{\text{particle}}c^2 = hf_1 + hf_2$$

Binding Energy

- The mass of a nucleus is less than the mass of its component nucleons
- This mass defect Δm is converted to energy, which is used to hold the nucleus together (the protons want to repel each other)
- Equivalent to the strong nuclear force (holding it together)
- The higher the mass defect / binding energy, the more stable the nucleus

$$\rightarrow 1u = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2 \text{ (for calculating in unified atomic mass units)}$$